For, let us set up an order among the classes, among the sub-classes in each class, and among the objects in each sub-class. Then from any given classification we may, by permutations of the objects which make changes in the order merely, but do not change the constituents of the classes and sub-classes—which, then, leave the classification as such the same—we may, by such operations, obtain various permutations of the totality of the rst objects. First, without changing the order of the classes or sub-classes, we may permute among themselves the objects in each sub-class. For any one sub-class this may be done in $r!$ ways, and, therefore, for all the st sub-classes in $(r!)^{st}$ ways. Next, we may permute among themselves the sub-classes in each class. For any one class this may be done in $s!$ ways; for all the t classes, then, in $(s!)^t$ ways. Finally, we may permute the classes in $t!$ ways.

Combining these various kinds of permutations, we see that each classification gives rise to $t!(s!)^t(r!)^{st}$ permutations of the rst objects. Since each of these latter permutations may be derived, by the process explained, from some classification or other, and two different classifications cannot give rise to the same permutation, it follows that the number of permutations of the rst objects is $t!(s!)^t(r!)^{st}$ times the number of classifications; that is, since there are $(rst)!$ permutations there are, as was to be shown, $(rst)!/t!(s!)^t(r!)^{st}$ classifications.

The argument may evidently be generalized to show (with a change in the notation) that

$$\frac{(r_1r_2\cdots r_n)!}{r_1!(r_2!)^{r_1}(r_3!)^{r_1r_2}\cdots(r_n!)^{r_1r_2\cdots r_{n-1}}}$$

is an integer.

228. Proposed by HERMON C. KATANIK,* Indianapolis, Ind.

Deduce a formula for the difference between any two squares, and thus show that (1) The difference between any two consecutive squares is of the form $2n + 1$; (2) The difference between any two squares is even or odd according to whether they are separated by an odd or even number of squares; (3) The differences of the squares of the consecutive terms of any arithmetic progression form another arithmetic progression.

SOLUTION BY WALTER C. EELLS, Whitman College.

Let T_i be the ith term. Then $T_n = n^2$, $T_{n+k} = (n+k)^2$.

$$T_{n+k} - T_n = (n+k)^2 - n^2 = (n+k-n)(n+k+n) = k(2n+k).$$

(1) If $k = 1$, $T_{n+1} - T_n = (2n+1)$.

(2) When separated by an odd number of squares $T_{n+2k'} - T_n = 2k'(2n+2k')$, which is even. When separated by an even number of squares $T_{n+2k'+1} - T_n = (2k'+1)(2n+2k'+1)$, which is odd.

(3) Since k is constant for any given arithmetic series, $T_{n+mk} - T_{n+(m-1)k}$ is constant and is the constant difference for another arithmetic series.

Also solved by HORACE OLSON and HERBERT N. CARLETON.

QUESTIONS AND DISCUSSIONS.

SEND ALL COMMUNICATIONS TO U. G. MITCHELL, University of Kansas, Lawrence, Kans.

A number of questions published in this department have been standing for some time without having been answered. We are re-publishing these in full this month in the hope that some of our readers may thereby be stimulated to send in suitable replies.

12. In view of the notation used by Professor Slobin in his "Note on Certain Algebraic Equations" published in the MONTHLY for April, 1914, pages 113–115, a discussion would be desirable as to the best notation for complex roots in general, and in particular for eliminating the conspicuous ambiguities introduced by the notation above cited.

15. We are in receipt of the following communication from Mr. W. E. Heal, of Washington, D. C.: "In the Proceedings of the Royal Society of Edinburgh, Vol. VII, p. 144, in some mathematical notes by Professor P. G. Tait, it is stated:

* Deceased since proposing this problem.

"'If $x^3 + y^3 = z^3$, then $(x^3 + z^3)^3y^3 + (x^3 - y^3)^3z^3 = (z^3 + y^3)^3x^3$.

"'This furnishes an easy proof of the impossibility of finding two integers the sum of whose cubes is a cube.'

"The writer has failed to see how this 'easy proof' follows and has been unable to find the question discussed or even mentioned in Tait's collected works. Can some reader of the MONTHLY supply the missing link or links?"

20. Some of our readers would like to have a simple account, without proofs, of just what has been accomplished toward the proof of the theorem that the equation $x^n + y^n = z^n$ is impossible in integers when $n > 2$.

21. For the diophantine equation

$$x^2 - y^3 = 17$$

there are known the following solutions:

$$\begin{array}{rrrrrrrrr} x = & 3, & 4, & 5, & 9, & 23, & 282, & 375, & 378661, \\ y = & -2, & -1, & 2, & 4, & 5, & 43, & 52, & 5234. \end{array}$$

One of our readers, who supplied the foregoing facts, desires to know the answers to the following questions: Are there other solutions of the given diophantine equation? How may all the solutions of this equation be found by a systematic procedure?

26. Why should not the nomenclature of mathematics be made uniform? For example, why call a circle a *portion of a plane* in elementary geometry and a *curve* in analytic geometry? Why call a sphere a *ball* at one time and a *surface* at another time? And so on through all the configurations of two- and three-dimensional geometry.

28. Is it possible to obtain $\int \cos \theta^2 d\theta$ without expanding $\cos \theta^2$? If it is not, can some interesting properties of this integral be determined by treating it as a special function?

30. A certain Normal University wishes to offer thirty-five hours of college mathematics for the benefit of high-school teachers. What should these courses be in order that, primarily, they may be of the greatest value to high-school teachers of mathematics and, secondarily, that they may furnish stimulus for a more extended pursuit of the subject?

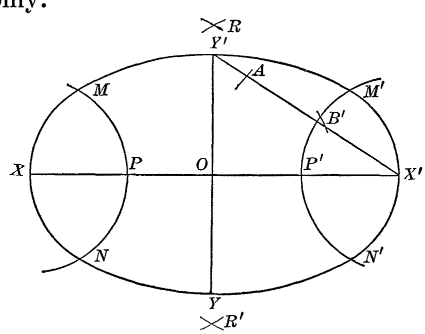
NEW QUESTION.

33. Under what conditions or to what extent is Mr. Iwerson's construction, given below, a useful or practical approximation to a true ellipse? What criterion can be given to measure definitely the degree of approximation?

AN APPROXIMATE CONSTRUCTION FOR AN ELLIPSE.

By RICHARD IWERSON, Everett, Washington.

Given the axes of an ellipse, to construct approximately the curve by use of ruler and compasses only.



Draw XX', the major axis, and YY', the minor axis, perpendicular to and bisecting each other at O. Draw the line $X'Y'$. With O as center and Ox as radius describe an arc cutting $X'Y'$ at B. With Oy as radius and X' as center